

# The Yang-Mills vacuum in Coulomb gauge<sup>1</sup>

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## Abstract

The Yang-Mills Schrödinger equation is solved in Coulomb gauge for the vacuum by the variational principle using an ansatz for the wave functional, which is strongly peaked at the Gribov horizon. We find an infrared suppressed gluon propagator, an infrared singular ghost propagator and an almost linearly rising confinement potential. Using these solutions we calculate the electric field of static color charge distributions relevant for mesons and baryons.

We report on the variational solution of the Yang-Mills Schrödinger equation in Coulomb gauge  $\partial_i A_i = 0$  performed in [1]. In this gauge the Yang-Mills Hamiltonian reads

$$H = \frac{1}{2} \int J^{-1}[A] \Pi J[A] \Pi + \frac{1}{2} \int B[A]^2 + \frac{g^2}{2} \int \rho(\hat{D}\partial)^{-1}(-\partial^2)(-\hat{D}\partial)^{-1}\rho(1)$$

Here  $J[A] = \det(-\hat{D}[A]\partial)$  is the Faddeev-Popov determinant,  $B[A]$  is the color magnetic field,  $\Pi(x) = \delta/i\delta A(x)$  is the momentum operator representing the color electric field and  $\rho(x) = -\hat{A}(x)\Pi(x)$  is the non-Abelian color charge. We use the following ansatz for the Yang-Mills wave functional

$$\Psi = \mathcal{N} J^{-\frac{1}{2}}[A] \exp\left(-\frac{1}{2} \int d^3x d^3x' A(x) \omega(x-x') A(x')\right), \quad (2)$$

where the kernel  $\omega(x-x')$  is determined by minimizing the vacuum energy

$$\langle \Psi | H | \Psi \rangle = \int DA J[A] \Psi^*[A] H \Psi[A]. \quad (3)$$

Thereby we restrict ourselves to 2-loop diagrams. Minimization of the energy gives rise to a set of coupled Schwinger-Dyson equations for the gluon propagator

$$\langle \Psi | A_i^a(x) A_j^b(x') | \Psi \rangle = \frac{1}{2} \delta^{ab} t_{ij}(x) \omega^{-1}(x-x'), \quad (4)$$

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<sup>1</sup>Invited talk given by H. Reinhardt at the Confinement conference, Sardinia 2004

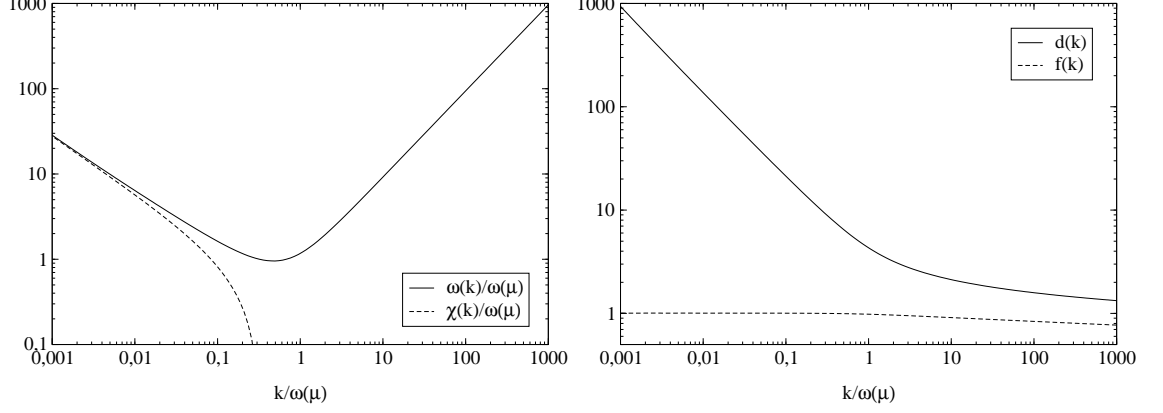


Figure 1: Solution for the gap function  $\omega(k)$  (left) and Ghost form function  $d(k)$  with Coulomb correction  $f(k)$  (right).

the ghost form factor  $d$  defined by

$$\langle \Psi | (-\hat{D}\partial)^{-1} | \Psi \rangle = \frac{d}{-\partial^2}, \quad (5)$$

the Coulomb form factor

$$\langle \Psi | (-\hat{D}\partial)^{-1} (-\partial^2) (-\hat{D}\partial)^{-1} | \Psi \rangle = \frac{d^2 f}{-\partial^2} \quad (6)$$

and the curvature in the space of gauge orbits

$$\chi = -\frac{1}{2} \frac{\delta^2 \ln J[A]}{\delta A \delta A}. \quad (7)$$

Resorting to the angular approximation the Schwinger-Dyson equations can be solved analytically in the infrared  $k \rightarrow 0$

$$\omega(k) = \chi(k) \sim \frac{1}{k}, \quad d(k) \sim \frac{1}{k}, \quad f(k) \rightarrow \text{const} \quad (8)$$

and in the ultraviolet  $k \rightarrow \infty$

$$\omega(k) \rightarrow k, \quad \frac{\chi(k)}{\omega(k)} \rightarrow 1/\sqrt{\ln k}, \quad d(k) \sim 1/\sqrt{\ln k}, \quad f(k) \sim 1/\sqrt{\ln k}. \quad (9)$$

Here the ghost form factor has been assumed to fulfil the so-called horizon condition  $d(k \rightarrow 0) \rightarrow \infty$ , but otherwise the above asymptotic behaviour is independent of the details of the renormalization. The numerical results are shown in figure 1. The gluon energy  $\omega(k)$  is infrared divergent signalling gluon confinement.

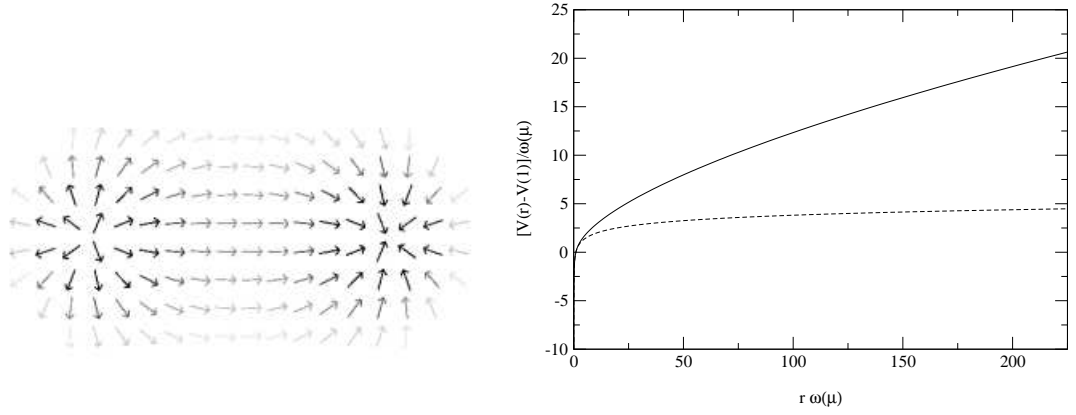


Figure 2: (left) Field lines of the longitudinal chromoelectric field of a charge-anticharge pair.  
(right) Coulomb Potential with (full line) and without inclusion of the curvature (dashed line).

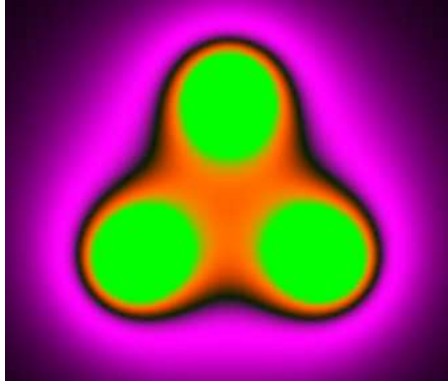


Figure 3: The magnitude of the longitudinal chromoelectric field of a three-quark color-singlet state.

The electric field generated by a static charge distribution  $\rho(x)$  is given by

$$E(x) = -\partial_x \int d^3x' \langle \Psi | \langle x | (-\hat{D}\partial)^{-1} | x' \rangle | \Psi \rangle \rho(x') . \quad (10)$$

In figure 2 we show the electric field generated by a static quark-antiquark pair. One observes the formation of a color flux tube between the static charges and accordingly we find an (almost) linearly rising confinement potential, see figure 2. Figure 3 shows the module of the static electric field induced by three static color charges in a color singlet state. The flux distribution seems to prefer a so-called Y-shape. Let us also mention that similar calculations, however, with a different ansatz for the wave functional and ignoring the curvature fully [2] or partly [3], have been carried out.

Recently, we have been able to show, that the above presented results do not depend on the detailed ansatz for the wave functional, but does crucially depend on the curvature  $\chi$  in the space of gauge orbits [4]. In particular, the infrared limit is uniquely determined and to 1-loop order the vacuum wave functional becomes  $\Psi[A] = 1$ .

The approach presented above is rather encouraging and calls for a more detailed study of the vacuum properties of Yang-Mills theory.

## References

- [1] C. Feuchter and H. Reinhardt, Phys. Rev. **D70**, (2004) 105021, hep-th/0408236
- [2] A.P. Szczepaniak and E.S. Swanson, Phys. Rev. **D65** (2002) 025012
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